# Engineering Notes

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## Lift Effect on Transonic Wind-Tunnel Blockage

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At transonic speeds, the flow disturbance extends laterally a great distance from a model. The interference of the wind-tunnel wall to the flow over the model is therefore much more severe than that at subsonic speeds. A unique characteristic of transonic flows is the nonlinear compressibility effect. Physically, the enlargement of the streamtube as the flow accelerates from subsonic to supersonic speeds over the body gives an impression that the body is locally thickened. When the airfoil is at lifting condition, the acceleration of the flow on the upper surface and deceleration of the flow on the lower surface further enhance this effect. For three-dimensional flow over a slender aircraft, the analysis of this nonlinear compressibility leads to the extended transonic equivalence rule with lift.[1]

For a two-dimensional flow past an airfoil in a wind tunnel, this apparent thickening of the airfoil induces additional blockage to the tunnel flow. In a recent analysis of the tunnel wall interference in transonic flows, the boundary value problem has been formulated in a systematic manner by a perturbation method.[2,3] The nonlinear compressibility appearing as an additional doublet was demonstrated in the numerical solutions of the boundary value problem.[2] In the development of the asymptotic solutions in the perturbation analysis[3] the nonlinear compressibility effect is delineated explicitly, leading to direct engineering applications. In the analysis within the framework of the transonic small disturbance theory, the tunnel wall interference is considered as a perturbation to the basic flow around the airfoil. As the ratio of two characteristic lengths defining the problem (namely, the airfoil chord and the tunnel height) approaches zero, the problem admits a singular perturbation treatment.

In the outer region, the perturbation potential $\phi$ can be developed as[3,4]

$$\phi\left(x,y,\frac{1}{H}\right)=\phi_0(x,y)+\frac{1}{H}\phi_1(x,y)+\frac{\log H}{H}\phi_2(x,y)+\ldots \quad (1)$$

where $x=\tilde{x}/H$ and $y=\tilde{y}/H$. $H$ is the half height of the wind tunnel and $\tilde{x}$, $\tilde{y}$ are the coordinates along and normal to the centerline of the tunnel, respectively. All variables are in the form of the transonic similarity transformation. The transonic small disturbance equation can then be written for each order as

$$K\phi_{0xx}+\phi_{0yy}=0 \quad (2a)$$

$$K\phi_{1xx}+\phi_{1yy}=(\gamma+1)\phi_{0x}\phi_{0xx} \quad (2b)$$

$$K\phi_{2xx}+\phi_{2yy}=0 \quad (2c)$$

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where $K$ is the transonic similarity parameter, $K=(1-M^2)/M\delta^{2/3}$; $M$ is the freestream Mach number; $\delta$ the thickness of the airfoil; and $\gamma$ the ratio of specific heats. For a wind tunnel with porous walls, the boundary conditions at the wall with porosity $P$ for each order are, respectively

$$\phi_{nx}\pm\frac{1}{P}\phi_{ny}=0, \qquad n=0,1,2,\ldots; \quad \text{at} \quad y=\pm H \quad (3)$$

By properly matching with the solution of the inner region, the inner expansion of the outer solution can be written as[3]

$$\phi\sim-\frac{\gamma_0}{2\pi}(\theta+a_1y)+\frac{\log H}{H}\left[\frac{\gamma+1}{4\pi^2K}\left(\frac{\gamma_0}{2}\right)^2\times\left(\frac{\cos\theta}{r}+\frac{b_2}{K}x\right)\right]+\frac{1}{H}\left[\frac{d_1}{2\pi\sqrt{K}}\left(\frac{\cos\theta}{r}+\frac{b_2}{K}x\right)+\frac{\gamma+1}{4\pi^2K}\left(\frac{\gamma_0}{2}\right)^2\frac{\log r}{r}\cos\theta-\frac{\gamma+1}{16\pi^2K}\left(\frac{\gamma_0}{2}\right)^2\frac{\cos 3\theta}{r}+\text{complementary functions}-\ldots\right] \quad (4)$$

The interference solutions are presented in $a_1$, $b_2$, and the complementary functions. The other terms are the singularities representing the airfoil, $\gamma_0$ is the circulation, and $d_1$ the doublet strength based on the cross-sectional area of the airfoil. The first term in Eq. (4) is the zero-order solution in the form of the classical linear theory of wind-tunnel wall interference, with the airfoil shrunk to a singular point represented by a vortex and yielding an angle-of-attack correction.[5] The first-order equation now includes the nonlinear compressibility effect at the right-hand side and the solution consists of three singularities. The first one is the solution of the homogeneous equation and is again obtained from the classical linear theory in the form of a doublet with the strength corresponding to the cross-sectional area of the airfoil. If the flow is dominated by lift, the particular integrals of the nonhomogeneous equation yield the next two terms. The corresponding complementary functions satisfying the wall boundary condition yield the interference potentials for the nonlinear compressibility correction. These functions can be derived by a linear method such as Fourier transform.[6] All these singularities represent the displacement effect of the airfoil causing blockage of the tunnel flow. The last two singularities do not vanish with the airfoil thickness as long as lift persists.

The expansion of the solution for nonlinear compressible flow includes logarithmic terms [see Eq. (1)]. The lowest order has the order of $\log H/H$ and the solution has the form of a doublet. By matching with the inner solution the strength of the doublet is obtained for lift-dominated flow.[3]

The magnitude of the singularities in the first-order terms can be estimated in comparison with the doublet strength $d_1$ based on the cross-sectional area of the airfoil. At a large distance from the airfoil, these singularities behave similarly to that of a doublet and approximated expressions can thus be derived for them. At $r\gg 1$, the first singularity approaches the limit

$$\frac{\log r}{r}\simeq\frac{1}{r} \quad (5)$$

**Table 1 Effective doublet strength**[a]

| | $\alpha$, deg | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $C_L$ | 0.215 | 0.370 | 0.550 |
| $\gamma_0$ | 0.375 | 0.645 | 0.958 |
| $d_e$ | 0.020 | 0.059 | 0.130 |
| $d_e/d_l$ | 0.031 | 0.091 | 0.200 |

[a] NACA 0012 airfoil; $\delta=0.12$; area $=0.0823$; $M_\infty=0.8$; $K=1.85$; $d_l=0.65$; $H=1.32$. (The physical tunnel half height is taken to be three times the airfoil chord length.)

The second singularity can be written as

$$\frac{\cos 3\theta}{r}=\frac{\cos\theta}{r}(4\cos^2\theta-3) \tag{6}$$

For $r \gg 1$, the interference of the wall is dominating only within a finite range at both sides of the vertical axis, i.e., $\theta \simeq \pm\pi/2$. Thus it may be approximated as

$$\frac{\cos 3\theta}{r}\simeq -\frac{3\cos\theta}{r}, \qquad r\gg 1, \qquad \theta\simeq\frac{\pi}{2} \tag{7}$$

With these approximations and the doublet of the logarithmic term, an effective doublet can be formed as

$$d_e=\frac{\gamma+1}{2\pi\sqrt{K}}\left(\frac{\gamma_0}{2}\right)^2\left[\frac{7}{4}+\log H\right] \tag{8}$$

The effective doublet is induced by the nonlinear compressibility of the flow. The strength of the doublet is proportional directly to the square of the circulation, hence the lift, and inversely to the square root of $K$, the transonic similarity parameter. At sufficiently high lift and with the freestream Mach number close to unity, the effective doublet strength becomes significant in comparison with $d_l$ corresponding to the geometrical area of the airfoil. This is shown in Table 1.

For an airfoil of 12% thickness and at a freestream Mach number of 0.8, even in moderate lift, the additional doublet strength amounts to 20% of the geometrical value.

To calculate the blockage of the tunnel flow, the boundary value problem of the linear homogeneous equations of Eqs. (2b and 2c) are solved for the singularities discussed previously. With the approximation of the singularities as shown in Eq. (8), the blockage solution can be obtained immediately from the linear theory similar to the first term of the order $1/H$ in Eq. (4). It shows that the blockage is directly related to the doublet strength. Thus the estimation of the effective doublet strength as shown in Table 1 applies directly to the blockage calculation. It should be noted that the present results are not restricted to porous walls as the singularities discussed are induced by the airfoil alone.

In summary, the perturbation analysis of wind-tunnel wall interference to the airfoil in transonic flows delineates an effective flow displacement due to lift as induced by the nonlinear compressibility correction. This effective flow displacement is significant in comparison with that due to the geometrical area of the airfoil, especially at high lift and a freestream Mach number close to unity. An approximated relation in the form of an effective doublet has been derived for this effect and can be applied directly in the blockage calculation.

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